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LETTER TO THE EDITOR

Collective pinning and the magnetization of the Bean critical state

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Abstract. The pinning force calculated from Larkin–Ovchinnikov collective pinning theory is used to determined the density profile (in one, two and three dimensions) of flux lines in the Bean critical state. In all cases one finds a result different from the conventional linear profile. The dependence of the corresponding magnetization on the external magnetic field is calculated. Strong single-particle pinning leads, in three dimensions, to a behaviour distinctly different from the collective pinning result.

The purpose of the present letter is to draw the attention to the point that measurements of the magnetic flux profile, and the magnetization in the Bean critical state, at low magnetic fields are able to provide a direct test of the collective pinning theory developed by Larkin and Ovchinnikov.

First some terminology. Consider a type-II superconductor cooled in zero external magnetic field to a temperature below the superconducting transition temperature. When an external field of strength H larger than the lower critical field H_{c_1} is applied, flux lines start to move into the bulk of the superconductor [1]. Inhomogeneities in the superconducting matrix will tend to pin the flux lines. The competion between the repulsive flux-line–flux-line interaction and the flux-line–pinning-centre interaction will determine the density profile of the flux lines when force equlibrium between the magnetic pressure (caused by the flux line repulsion) and the pinning force is eventually achieved. The temperature is assumed to be sufficently low to allow thermal activation to be neglected. This is the Bean critical state [2]. The density profile of the flux lines is identical to the profile of the magnetic induction inside the sample from which the magnetization of the sample is easily calculated.

Consider a slab geometry with the external field parallel to the surface of the sample. The force balance in the Bean critical state can be written as

$$\left|\frac{B(x)}{4\pi}\frac{\mathrm{d}B(x)}{\mathrm{d}x}\right| = F_{\mathrm{p}}. \tag{1}$$

Here $B(x)$ denotes the magnetic induction at position x inside the sample and F_{p} is the pinning force per unit volume. (The induction is related to the flux-line density n_{v} through $B = N_{\mathrm{v}}\Phi_0$, where Φ_0 is the flux quantum [1].) The pinning force is connected to the critical current, J_{c}, through the expression for the Lorentz force, namely, $F_{\mathrm{p}} = BJ_{\mathrm{c}}/c$ (c is the velocity of light). In the traditional treatmeant of the Bean critical state, one assumes that J_{c} is a space- and field-independent constant when one solves equation (1) [1]. This leads immediately to the solution

$$B(x) = H\left(1 - \frac{x}{\Lambda}\right). \tag{2}$$

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Here H is the external field and $\Lambda = cH/4\pi J_c$. When one substitutes this expression for the induction into the expression for the magnetization

$$M = \frac{1}{8\pi d}\int_0^d (B(x) - H)\,\mathrm{d}x \tag{3}$$

one obtains for fields larger than the penetration field, H_p, the well know result $M = dJ_c/2\pi c$ [1]. A field dependence of M is conventionally obtained by including, at this level, a field dependence of J_c obtained from some pinning model [3]. This is clearly a somewhat arbitrary way to determine the field dependence of M. One should rather solve equation (1) including the field dependence of F_p directly. This is what we are going to do below.

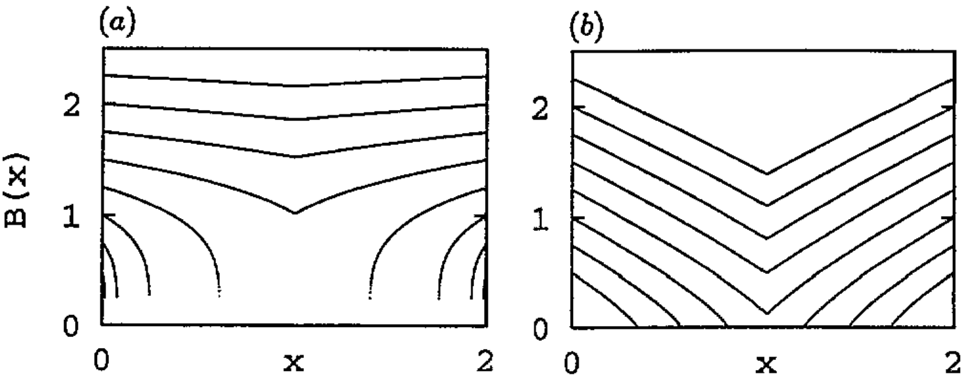

Figure 1. A slab of thickness equal to $2d$ is placed in an external magnetic field parallel to the surface of the sample (x is measured in units of d). The profile of the magnetic induction is sketched for different values of the external magnetic field. (a) corresponds to $\alpha = 1/4$ (collective pinning with shear and tilt). (b) corresponds to $\alpha = 4/5$ (amorphous lattice limit).

Tang [4] and Barford, Beere, and Steer [5] have presented simulations of one-dimensional particle models for which the linear density profile (obtained from the assumption that F_p is proportional to the particle density) does not apply. Both simulations find a parabolic profile

$$B(x) \sim \left(1 - \frac{x}{\Lambda}\right)^{1/2} \tag{4}$$

where Λ is the distance the pinning potential allows the particles—or flux lines—to penetrate into the sample. This behaviour is obtained from equation (1) if one assumes F_p to be independent of the density. Tang has shown that a density-independent pinning force is in fact obtained from the Larkin–Ovchinninkov (LO) theory of collective pinning [6] applied to one dimension. However, as we will see below, the result for collective pinning is, in one dimension, the same as the result for strong single-particle pinning. Fortunately, in higher dimensions the two cases lead to distinguishable behaviour of the density profile.

Below we derive the form of $B(x)$ using elasticity theory. We apply the theory to two- and three-dimensional flux-line systems. We find that the induction in an infinite slab geometry, see figure 1, varies with distance x from the surface as

$$B(x) = H\left(1 - \frac{x}{\Lambda}\right)^{\alpha} \tag{5}$$

where the length scale Λ is given below [7].

The exponent depends on dimension and on the type of elastic deformations induced by the pinning potential. In one dimension $\alpha = 1/2$, in two dimension $\alpha = 1/2$ for an incompressible flux system. If compression is relevant one finds that $\alpha = 2/5$. In three dimensions the value for a flux system which only allows shear and tilt deformations is $\alpha = 1/4$. When compression is included the exponent changes to $\alpha = 1/5$. For an amorphous flux lattice $\alpha = 4/5$. In the limit of strong single-particle pinning $\alpha = 1/2$ in any dimension. The corresponding magnetization, M, varies for fields smaller than the field at which the sample becomes fully penetrated as $M \sim H^{1+1/\alpha}$ and for fields $H > H_p$ as $M \sim H^{1-1/\alpha}$.

We now describe how these results are derived from equation (1) by assuming that F_p can be calculated from the LO theory of collective pinning [6].

The LO collective pinning theory consists of two assumptions. First one assumes that the total pinning force acting on a certain correlated subvolume, V_c, of the flux system is given by the fluctuations in the sum of the forces from the active pinning centres within the volume V_c. The probability for a centre to be active, i.e. to exert a force on the flux lattice is given by the probability of a flux line to be within the range R_p of the pinning centre. Hence, only the fraction $n_v R_p^2$ of the total number of $V_c n_p$ pinning centres are active (n_p denotes the number of pinning centres per unit volume). The fluctuations in the sum of these pinning centres are given by

$$(V_c n_p n_v R_p^2 \langle f_0^2 \rangle)^{1/2}. \tag{6}$$

Here $\langle f_0^2 \rangle$ denotes the fluctuations in the force exerted by one pinning centre on one flux line. $\langle f_0^2 \rangle$ is independent of n_v [8]. The pinning force per unit volume is

$$F_p = \left(n_p n_v R_p^2 \langle f_0^2 \rangle / V_c\right)^{1/2}. \tag{7}$$

The second assumption concerns the determination of V_c. One assumes that the correlated volume is given by optimizing the competition between the elastic strain energy induced by the relaxation to the pinning centres within the volume V_c and the gain in pinning energy obtained due to the relaxation [6]. The strain of the induced elastic deformations is estimated by R_p/L where L is one of the linear dimensions of the volume V_c. Let us consider the most general three-dimensional case. The total energy per unit volume of the volume V_c is given by

$$E = \tfrac{1}{2} C_{11} \left(R_p/R_\parallel\right)^2 + \tfrac{1}{2} C_{66} \left(R_p/R_\perp\right)^2 + \tfrac{1}{2} C_{44} \left(R_p/L_c\right)^2 - R_p F_p. \tag{8}$$

The first three terms correspond to the compression, shearing and tilting of the volume $V_c = R_\parallel R_\perp L_c$. C_{11} is the compression modulus, C_{66} the shear modulus, and C_{44} the tilt modulus. The density dependence of the elastic coefficients for small fields is given by $C_{11} \sim n_v^2$, $C_{44} \sim n_v^2$, and $C_{66} \sim n_v$ [9].

One determines V_c by minimizing E with respect to $R_\parallel$, $R_\perp$, and L_c. The expression for the pinning force is obtained by substituting the value for V_c found from this minimalization into equation (7). We write the result as $F_p = A/n_v^\beta$. Here A is a factor independent of the flux-line density. A will of course depend on material properties, dimension and the type of relevant elastic deformations. The value of β depends (see below) on dimension and on the type of deformations induced by the pinning centres. The induction is immediately found to be of the form given in equation (5) with $\alpha = 1/(2+\beta)$ and $\Lambda = \alpha H^{1/\alpha}/4\pi A$. The magnetization is found from equation (3) to behave as

$$M = \frac{H}{4\pi} \left(\frac{\alpha}{\alpha+1} \frac{1}{4\pi A d} H^{1/\alpha} - 1 \right) \tag{9}$$

for $H < H_p$. For fields above the penetration field one has

$$M = \frac{H}{4\pi}\left\{\frac{\alpha}{\alpha+1}\frac{1}{4\pi Ad}H^{1/\alpha}\left[1-\left(1-\frac{4\pi Ad}{\alpha H^{1/\alpha}}\right)^{1+\alpha}\right]-1\right\} \tag{10}$$

The expression in equation (10) behaves for $H > (4\pi Ad/\alpha)^\alpha$ as $M \sim H^{1-1/\alpha}$.

We now discuss the solution of equation (1) in different regimes. Table 1 contains the values of the the exponent α for dimensions one, two, and three and for different types of deformations of the flux system. The solution for $B(x)$ and $M(H)$ is shown in figures 1 and 2 for different values of α.

Table 1. The exponent α of the density profile given by equation (3) for different types of elastic distortions of the flux system. A precise definiton of the the different cases is given in the text after equation (7).

Dimensions	Compression	Shear	Shear + compression	Shear + tilt	Shear + tilt + compression	Amorphous lattice limit	Strong single-particle pinning
1	1/2	—	—	—	—	—	1/2
2	—	1/2	2/5	—	—	—	1/2
3	—	—	—	1/4	1/5	4/5	1/2

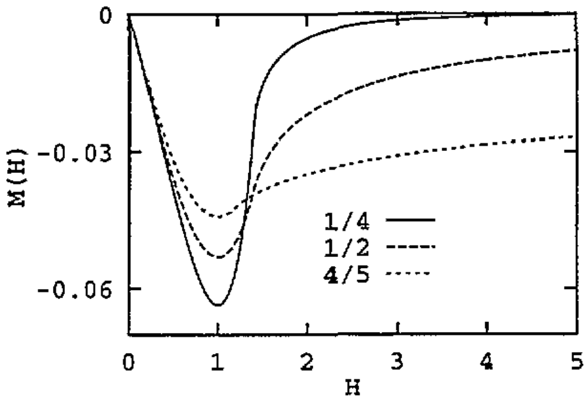

Figure 2. Magnetization, as given by equations (6) and (7), for the values $\alpha = 1/4,\ 1/2,\ 4/5$, relevant to three dimensions. We have put $4\pi Ad = 1$.

Equation (8) describes a three-dimensional system for which shear, tilt, and compression are equally relevant. The correlated volume is given by $V_c = R_\perp R_\parallel L_c$. The exponent β is found to be equal to 3 which gives $\alpha = 1/5$.

The compression modulus is often much larger than the shear and tilt moduli. This is in particular the case for homogeneous deformations of the flux-line lattice. In this case it is more appropriate to exclude the term proportional to C_{11} from equation (8). The correlated volume is then given by $V_c = R_\perp^2 L_c$. One finds $\beta = 2$ and $\alpha = 1/4$.

As the strength of the pinning centres increases $R_\perp$ decreases. When $R_\perp$ becomes smaller than the average distance, $a_0 = 1/\sqrt{n_v}$, between the flux lines the correlated volume becomes equal to $V_c = a_0^2 L_c$. In this limt L_c is determined from equation (8) by neglecting the compression and the shear term. One finds $\beta = -3/4$ and $\alpha = 4/5$.

Finally we consider the case of strong pinning centres which are all able to overcome the elastic restoring forces and therefore pin with a maximum pinning force f_0. The volume pinning is in this case $F_p = n_p f_0$. That corresponds to $\beta = 0$ and $\alpha = 1/2$. This result is obviously independent of dimension.

To apply equation (8) to two dimensions one neglects the term describing the tilt. The correlated volume in the case where $C_{11} \simeq C_{66}$ is given by $V_c = R_\perp R_\parallel$. The exponent β is found to be equal to 1/2 and $\alpha = 2/5$. If $C_{11} \gg C_{66}$ we have $V_c = R_\parallel^2$. The corresponding exponents are $\beta = 0$ and $\alpha = 1/2$.

In one dimension only compression can occur. The correlated volume is given by $V_c = R_\perp$. The length scale $R_\perp$ is found from equation (8) by leaving out the terms proportional to C_{66} and C_{44}. One finds $\beta = 0$ and $\alpha = 1/2$.

The Bean critical state hypothesis [2] as given by equation (1) was combined with the Larkin-Ovchinnikov theory of collective pinning [6] to determine the flux profile in the Bean critical state. An algebraic profile

$$B(x) \sim \left(1 - \frac{x}{\Lambda}\right)^\alpha \qquad (11)$$

with $\alpha < 1$ was found in all cases.The prediction of the theory in one dimension is in agreement with recent simulation results [4, 5].

Low-field decoration experiments or magnetization measurements on three dimensional samples should easily be able to distinguish between the predictions of the collective pinning theory ($\alpha = 1/4$, only shear and tilt assumed to be relevant), the linear Bean profile ($\alpha = 1$), and the result for strong single-particle pinning ($\alpha = 1/2$).

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[7] Since $\partial B/\partial x = J_c/c$ according to equation (1) the functional form of $B(x)$ in equation (5) leads to a diverging critical current as $x \to \Lambda$. This only means that as the density of flux lines decreases the relation $\partial B/\partial x = J_c/c$ loses its meaning. The interaction between the flux lines vanishes as $B(x) \to 0$ and the flux lines are pinned individually by a force smaller than or equal to the maximum of the individual pinning force. This implies that a cross-over to the strong single-particle pinning occurs. Hence J_c remains finite.
[8] This statement is only true for low magnetic fields. As the upper critical field, H_{c_2}, is approached one needs to take into account that the interaction strength between one pinning centre and one flux line is reduced; approximately by a factor $1 - H/H_{c_2}$. This reduction occur due to the depletion of the superconducting condensate. See, for instance,
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We use the dependence of the elastic moduli on the density of flux lines in the low-density range (neglecting the wavevector dependence of the elastic moduli). The reason is that our results are experimentally most accessible in this range. Moreover, we do not expect the estimate of the pinning force to be applicable

for fields larger than about half the value of the upper critical field. At larger fields we expect plastic deformations (ignored in equation (8)) to be relevant. See [10].
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